Confirming an Integral Converges

Use limit comparison to show that $\displaystyle\int_1^\infty \frac{dx}{(5x+2)^2}$ is finite.

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$$\int_1^{\infty} \frac{1}{(5x+2)^2}\,dx$$

$$= \int_1^{\infty} \frac{1}{25x^2+20x+4}\,dx$$

As $x \to \infty$, $\frac{1}{25x^2+20x+4} \cong \frac{1}{25x^2}$.

$$\lim_{N\to\infty} \int_1^N \frac{1}{25x^2}\,dx$$

$$= \frac{1}{25}\left[-\frac{1}{x}\right]_1^{\infty}$$

$$= \frac{1}{25}(0+1)$$

$$= \frac{1}{25}$$

$\therefore \int_1^{\infty} \frac{1}{25x^2}\,dx = \frac{1}{25}$

$\therefore \int_1^{\infty} \frac{1}{(5x+2)^2}\,dx$ is also a finite value.